Problem 25.33

An electrical potential field is defined as:

$$V = a + bx$$

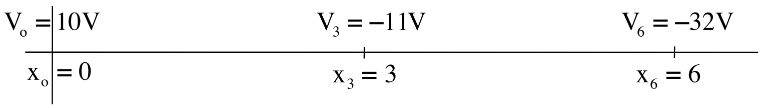
where a = 10 volts and b = -7 volts (so $V = 10 - 7x$).

a.) What is V(x = 0), V(x = 3 m) and V(x = 6 m)?

$$V(x=0) = 10 \text{ volts}$$

$$V(x = 3\text{ m}) = (10\text{ volts}) - 7(3\text{ m}) \\ = -11\text{ volts}$$

$$V(x = 3\text{ m}) = (10\text{ volts}) - 7(6\text{ m}) \\ = -32\text{ volts}$$



The first thing to notice about the graphical representation of the voltages is that they are getting smaller as you move along the +x axis. As voltages proceed *downhill*, so to speak, relative to the electric field that created them, the electric field must be in the +x direction.

The second thing to notice is that as one were to proceed in the –x direction, sooner or later you would run into a point where the voltage was ZERO. This would NOT mean the electric field was zero at that point. The electric field is equal to the *rate at which the electrical potential is CHANGING*. As long as the voltage is changing, there must be an electric field present.

b.) What is the electric field at the points highlighted?

We need an electric field function that is good for any *x*, so I'm going to do this the formal way using what is called a *del operator*. Specifically:

Starting with the shorthand notation for the *del operator* acting on a voltage function, then expanding, we have:

$$\vec{E} = -\vec{\nabla} V$$

$$\Rightarrow \quad \vec{E} = -\left[\left(\frac{\partial}{\partial x}\right)\hat{i} + \left(\frac{\partial}{\partial y}\right)\hat{j} + \left(\frac{\partial}{\partial z}\right)\hat{k}\right]V$$

$$\Rightarrow \quad = -\left[\left(\frac{\partial V}{\partial x}\right)\hat{i} + \left(\frac{\partial V}{\partial y}\right)\hat{j} + \left(\frac{\partial V}{\partial z}\right)\hat{k}\right]$$

$$\Rightarrow \quad = -\left[\left(\frac{\partial[(10-7x)]}{\partial x}\right)\hat{i} + \left(\frac{\partial[(10-7x)]}{\partial y}\right)\hat{j} + \left(\frac{\partial[(10-7x)]}{\partial z}\right)\hat{k}\right]$$

$$\Rightarrow \quad \vec{E} = -\left[\ -7\hat{i} \quad + \quad 0\hat{j} \quad + \quad 0\hat{k}\ \right]$$

$$\Rightarrow \quad \vec{E} = 7\hat{i}\ \text{V/m}$$

Apparently, the electric field is constant and, as such, the same everywhere. Note also that the direction is the same direction we concluded must be the case back in Part a.